COMPLETING THE SQUARES AND FORMULA METHODS

Some quadratic equations cannot be solved by factorisation method. In such cases, the method of completing the squares could be used. This method involves making the expression of the equation a perfect square which can then be factorised.

Example: Solve the equation $2x^2 + 3x - 2 = 0$ using the method of completing the squares.

Solving

$2x^2 + 3x - 2 = 0$

Step I: make the coefficient of x^2 unity by dividing through by the coefficient of x^2 i.e. 2 here

$$\frac{2x^2}{2} + \frac{3x}{2} - \frac{2}{2} = 0$$

$$x^2 + \frac{3}{2}x - 1 = 0$$

Step II: if the RHS of the equation is not zero, we arrange it to make it zero.

$$x^2 + \frac{3}{2}x - 1 = 0$$

Step III: transfer the constant term to the RHS (i.e. 1 here); remember to change the sign.

$$x^2 + \frac{3}{2}x = 1$$

Step IV: make the LHS a perfect square by adding the square of half of the coefficient of x to both sides of the equation.

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = 1 + \left(\frac{3}{4}\right)^2$$

$$x^2 + \frac{3}{2}x + \frac{9}{16} = 1 + \frac{9}{16} = \frac{25}{16}$$

Step V: factorize the LHS

$$(x+\frac{3}{4})^2 = \left(\frac{5}{4}\right)^2$$

Step VI: make the LHS a perfect square by adding the square of half of the coefficient of x to both sides of the equation.

$$x+\frac{3}{4} = \pm\sqrt{\left(\frac{5}{4}\right)^2} = \pm\frac{5}{4}$$

$$x = -\frac{5}{4}-\frac{3}{4}\ or\ \frac{5}{4}-\frac{3}{4}$$

$$x = -\frac{8}{4} or \frac{2}{4} = -2\ or \frac{1}{2}$$

$$\boldsymbol{x = -2\ or \frac{1}{2}}$$

The Quadratic Formula (Almighty Formula)

We could derive the almighty formula from a typical quadratic equation: $ax^2+bx+c=0$; where a≠0, through the method of completing the squares.

$$ax^2+bx+c=0$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$x^2+\frac{b}{a}x=-\frac{c}{a}$$

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 = -\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$\left(x+\frac{b}{2a}\right)^2 = -\frac{c}{a}+\left(\frac{b}{2a}\right)^2$$

$$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$

$$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \left(\frac{b}{2a}\right)^2}$$

$$x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$