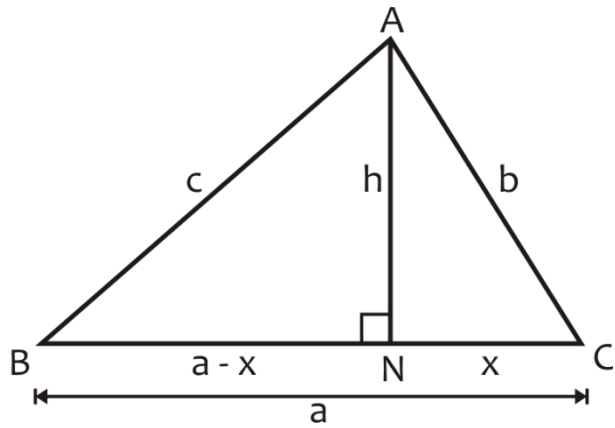
THE COSINE RULE

Given: any ΔABC (acute-angled and obtuse-angled triangles)



To prove: $c^2 = a^2+b^2 - 2abcosC$

Construction: Draw the perpendicular from A (vertex) to BC.

Proof:

$$c^2 = (a-x)^2 + h^2 = a^2 - 2ax + x^2 + h^2 \text{ -- i}$$

$$b^2 = x^2 + h^2 \quad \text{--- ii}$$

Substituting the value of b^2 in eqn. i

$c^2 = a^2 - 2ax + b^2 = a^2 + b^2 - 2ax$ --- iii

To remove x from eqn. iii we use the trigonometric equation in relation to ∠x

$\cos C = \frac{x}{b}; \quad x = b\ cosC$ --- iv

Substitute the value of x into eqn. iii

$c^2 = a^2 + b^2 - 2abcosC$ **(proven)**

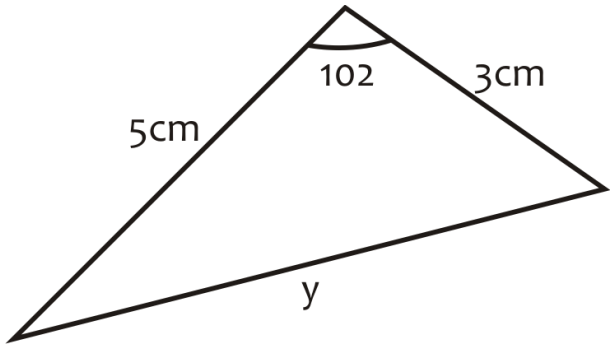
In either case, $c^2 = a^2 + b^2 - 2ab\ cosC$

similarly, $b^2 = a^2 + c^2 - 2ac\ cosB$

and $a^2 = b^2 + c^2 - 2bc\ cosA$

The formula, the cosine rule, is used for solving triangles which are not right-angled in which two sides and the include angle are given.

Example: Find *y* in the triangle below



Solving

$$y^2 = a^2 + b^2 - 2ab\ cosY$$

$$= 5^2 + 3^2 - 2 \times 5 \times 3 \times cos102$$

$$= 25 + 9 - 10 \times 3$$

$$\times (-\cos(180 - 102))$$

$$= 34 - 30(-cos78) = 34 + 30cos78$$

$$= 34 + 30 \times 0.20709 = 34 + 6.237$$

$$= 40.237$$

$$y = \sqrt{40.237} = \mathbf{6.34\ cm}$$